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Application of Minimum Cost Flow Problem: A Case Study of Crown Distributors in Kegalle, Sri Lanka

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Abstract—Every business organization's main objective is to maximize profit and satisfy its customers. Since business are an integral part of our environment, their objectives will be limited by certain environment factors and economic conditions.This study was carried out to seek and arrive at the optimal distribution pattern of a distribution agency.Demand of customers, warehouse capacities and factory capacities were used as input data for the model. Firstly, we propose a linear programming approach to determine the optimal distribution pattern in order to minimize the total distribution cost. The proposed linear programming model is solved by standard simplex algorithm and Excel-solver software.It is observed that the proposed linear programming model is appropriate for finding the optimal distributing pattern and the total minimum cost. A case study is carried out in the Crown distributors, Kegalle.

Index Terms—Business, Crown distributors, distribution, linear programming, minimize, minimum cost flow, simplex algorithm.

1 Introduction

Everywhere we look in our lives, networks are apperent. Computer networks, highways, telecommunication networks, water delivery systems, and many others, are familiar to all of us. In each of these problem setting, we often wish to send some good(s) from one point to another, typically as efficiently as possible-that is, along a shortest path or via some minimum cost flow pattern. Network optimization has always been a core problem domain in operations research, computer science, applied mathematics, and many other fields of engineering, science, and management. The many applications in these fields not only occur "naturally" on some transparent physical network, but also in situations that apparently are quite unrelated to networks.

Minimum cost flow problem is the most fundamental of all network flow problems. Minimum cost flow problems arise in almost all industries, including communications, agriculture, manufacturing, transportation, healthcare, retailing, education, energy and medicine. The problem is easy to state: we wish to determine a least cost shipment of a commodity through a network in order to satisfy demands at certain nodes from available supplies at other nodes. This model has number of familiar and less familiar applications: the distribution of a product from plants to warehouses, or from warehouses to customers; the routing vechicles through a street network and etc.

Many researchers have examined the task of minimum cost flow. Methods of its solutions can be divided into graph techniques & the methods of linear programming. Ford & Fulkerson developed a classical and still frequently used method for solving this problem is the primal-dual method. This algorithm based on the theory of linear programming.

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Jewel, Busaker and Grown independently developed the successive shortest path algorithm. These researchers showed how to solve the minimum cost flow as a sequence of shortest path problem with arbitrary arc lengths. Edmonds and Karp independently observed that if the computations use vertex potentials, it is possible to implement these algorithms so that the shortest path problems have non-negative arc length. Minty and Fulkerson independently developed the out-of-kilter algorithm. Aashtiani and Magnanti have described an efficient implementation of this algorithm. The cycle-canceling algorithm is credited to Klein. Bertsekas and Tseng developed the relaxation algorithm and conducted extensive computational investigation of it. Grigoriadis, Kennington and Wang have described an efficient implementation of the relaxation and the network simplex algorithm.

A large number of real-world applications can be modeled by using minimum cost network flows with multiple objectives. Damian and Garrett (1991a) in their PhD. thesis work entitled the Minimum Cost Flow Problem and the Network Simplex Solution Method" in Ireland distribution network has Dublin and Belfast as supply nodes, while Cork, Galway, Limerick and Waterford were demand nodes. The Spanning tree technique was used to find the optimal solution. Further, according to Damian and Garrett (1991b) and Dantzig et al. (1950) first studied maximumflow and minimum-cut problem. They left their finding at that stage until mid 1950's. Goldberg and Tarjan (1990) dealt with nine different problem types, with the stated goal of aiding the third of Stalin's five-year plans to obtain the greatest possible usage of existing industrial resources. Principle among these problems were: distribution of work among individualmachines; distribution of orders among enterprises; distribution of raw materials; fuel and factors of production; minimization of scrap; and best plan of freight shipments. Shigeno et al. (2000), discussed various algorithms. He explained how efficient Edmond- Karp and

Push-relabel algorithms are suitable maximum flow problems.

In this article, minimum cost flow problem has been dealt when the supply, the demand and the distribution cost per unit of the quantitiy are known. A case study is carried out in the Crown distributors, kegalle; Sri Lanka. The problem has been posed as a linear programming problem and solve using Simplex algorithm and Excel-solver.

In this paper, second section gives the basic definitions and formation of the model, the area of the study is explained in third section, the fourth section elaborates the application of the model to the proposed case study. Solution of the problem is discussed in the fifth section and finally the conclusions are given in the sixth section of the paper.

2 Model Formation

2.1 Folw Models of Real Network

Real networks can be modelled as a directed graph $G(V,E)$, where V is the set of vertices($|V| = n$) and E is the set of directed arcs. Every arc $(i,j) \in X$ is associated with a set of non-negative values, reffered to as arc attributes of a network. Examples of arc attributes in a network are arc cost, distance, time, brandwith and etc. One of the important parts in the operation of a network is routing. Routing can be thought as sending goods from one vertex (source-s) to another vertex (sink-t)in a network. A routing task consists of finding a path $Pa(s,t)$ suitable to a given application that has end-to-end constraints. In the next section, we will present the minimum-cost flow problem, it says importance role in network routing algorithm.

2.2 Minimum Cost Flow Problem

Consider a network $G = (V,E)$ with $|V| = n$, and let $b \in R^n$. Here, b_i denotes the amount of flow that enters or leaves the network at vertex $i \in V$. If $b_i > 0$, we say that i is a source suppling b_i units of flow. If $b_i < 0$, we say i is a sink with a demand of $|b_i|$ units of flow. Further, let c_{ij} denotes the cost associated with one unit of flow on edge $(i,j) \in E$, and l_{ij} and u_{ij} respectively denote lower and upper bounds on the flow across this edge. The minimum-cost flow problem then asks for flow x_{ij} that conserve the flow at each vertex, repect the upper and lower bounds, and minimize the overall cost. The single commodity, linear minimum-cost network flow problem is defined as,

$$\min c_{ij}x_{ij}$$

subject to

$$\sum_{(i,j)\in E} x_{ij} - \sum_{(j,i)\in E} x_{ji} = b_i \quad \forall i \in V, \qquad (2.1)$$

$$l_{ij} \le x_{ij} \le u_{ij} \qquad (2.2)$$

We refer to (2.1) as flow conservation constraints and as capacity constraints. We assumed that all data are integer and wish to find an integer-valued optimal solution. Without loss of generality, we may further assume that all arc capacities are finite, all arc costs are non-negative, and the problem has a feasible solution. It also implies that $\sum_{i\in V} b_i = 0$.

3 Area of Study

The Crown distributors company in Kegalle is situated in the Sabaragamuwa province of Sri Lanka. It has two factories. In addition, it has four warehouses with storage facilities. The company sells its product to six customers C1, C2, C3, C4, C5, and C6.Customers can be supplied either from warehouse or from the factory direct (see figure 3.1).

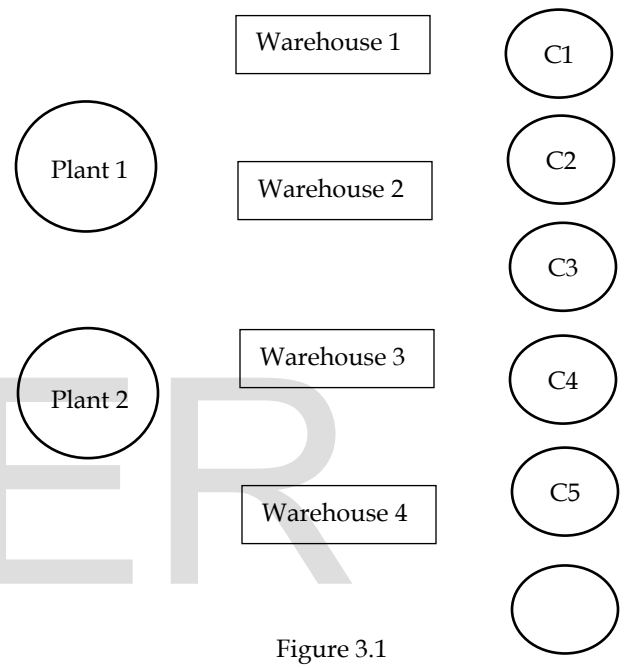

Figure 3.1

The distribution costs are known; they are given in table 3.1. (A dash indicates the impossibility of certain suppliers for certain warehouses or customers)

Table 3.1

Supp. to	Supplier					
	P1	P2	W1	W2	W3	W4
W1	0.5	-				
W2	0.5	0.3				
W3	1.0	0.5				
W4	0.2	0.2				
C1	1.0	2.0	-	1.0	-	-
C2	-	-	1.5	0.5	1.5	-
C3	1.5	-	0.5	0.5	2.0	0.2
C4	2.0	-	1.5	1.0	-	1.5
C5	-	-	-	0.5	0.5	0.5
C6	1.0	-	1.0	-	1.5	1.5

Each factory has a capacity (annual) given below which cannot be exceeded:

Plant 1 150 000

Plant 2 200 000

Each warehouse has a maximum (annual) throughput given below which cannot be exceeded:

Warehouse 1	70 000
Warehouse 2	50 000
Warehouse 3	100 000
Warehouse 4	40 000

Each customer has a monthly requirement given below which must be met:

C1	50 000
C2	10 000
C3	40 000
C4	35 000
C5	60 000
C6	20 000

4 Application

The factories, warehouses and customers will be numbered as below:

Factories: 1, 2
Warehouses: 1, 2, 3, 4
Customers: 1, 2, 3, 4, 5, 6

4.1 Decision Variables

The description of decision variables used to build the model is given below:

x_{ij} = quantity sent from factory i to warehouse j,
$i = 1, 2$, $j = 1, 2, 3, 4$;

y_{ij} = quantity sent from factory i to customer k,
$i = 1, 2$, $k = 1, 2, 3, 4, 5, 6$;

z_{ij} = quantity sent from warehouse j to customers k,
$j = 1, 2, 3, 4$, $k = 1, 2, \ldots, 6$.

There are 44 such variables.

4.2 Objective Function

The decision marker has to determine a least cost shipment of a commodity through a network. Thus the objective function is formulated as follows;

$$\sum_{\substack{i=1\\j=1}}^{\substack{i=2\\j=4}} c_{ij}x_{ij} + \sum_{\substack{i=1\\k=1}}^{\substack{i=2\\k=6}} d_{ik}y_{ik} + \sum_{\substack{j=1\\k=1}}^{\substack{j=2\\k=6}} e_{jk}z_{jk}, \qquad (4.1)$$

where the coefficients $c_{ij}, d_{ik,}$ and e_{jk} are given in the table 3.1.

4.3 Constraints

1. Factory Capacities

$$\sum_{j=1}^{2} x_{ij} + \sum_{k=1}^{6} y_{ik} \le \text{capacity}, \quad i = 1, 2. \qquad (4.2)$$

2. Quantity into Warehouses

$$\sum_{i=1}^{2} x_{ij} \le \text{capacity}, \quad j = 1, 2, 3, 4. \qquad (4.3)$$

3. Quantity out of Warehouses

$$\sum_{k=1}^{6} z_{jk} = \sum_{i=1}^{2} x_{ij}, \quad j = 1, 2, 3, 4. \qquad (4.4)$$

4. Customer Requirements

$$\sum_{i=1}^{2} y_{ik} + \sum_{j=1}^{4} z_{jk} = \text{requirement}, \; k = 1, 2, \ldots, 6. \qquad (4.5)$$

5 Results and Discussion

The solutions of the model for minimum cost flow by using standard simplex algorithm and Ms Excel solver are represented in table 5.1.(*The value 1000 indicatesthe impossibility of certain suppliers for certain warehouses or customers.)

Table 5.1

Crown Distributors, Kegalle, Sri Lanka

From		To		Flow	Unit Cost
P1	Plant 1	W1	Warehouse 1	0	0.5
P1	Plant 1	W2	Warehouse 2	0	0.5
P1	Plant 1	W3	Warehouse 3	0	1
P1	Plant 1	W4	Warehouse 4	40000	0.2
P1	Plant 1	1	Customer 1	50000	1
P1	Plant 1	2	Customer 2	0	1000
P1	Plant 1	3	Customer 3	0	1.5
P1	Plant 1	4	Customer 4	0	2
P1	Plant 1	5	Customer 5	0	1000
P1	Plant 1	6	Customer 6	20000	1
P2	Plant 2	W1	Warehouse 1	0	1000

P2	Plant 2	W2	Warehouse 2	50000	0.3
P2	Plant 2	W3	Warehouse 3	55000	0.5
P2	Plant 2	W4	Warehouse 4	0	0.2
P2	Plant 2	1	Customer 1	0	2
P2	Plant 2	2	Customer 2	0	1000
P2	Plant 2	3	Customer 3	0	1000
P2	Plant 2	4	Customer 4	0	1000
P2	Plant 2	5	Customer 5	0	1000
P2	Plant 2	6	Customer 6	0	1000
W1	Warehouse 1	1	Customer 1	0	1000
W1	Warehouse 1	2	Customer 2	0	1.5
W1	Warehouse 1	3	Customer 3	0	0.5
W1	Warehouse 1	4	Customer 4	0	1.5
W1	Warehouse 1	5	Customer 5	0	1000
W1	Warehouse 1	6	Customer 6	0	1
W2	Warehouse 2	1	Customer 1	0	1
W2	Warehouse 2	2	Customer 2	10000	0.5
W2	Warehouse 2	3	Customer 3	0	0.5
W2	Warehouse 2	4	Customer 4	35000	1
W2	Warehouse 2	5	Customer 5	5000	0.5
W2	Warehouse 2	6	Customer 6	0	1000
W3	Warehouse 3	1	Customer 1	0	1000
W3	Warehouse 3	2	Customer 2	0	1.5
W3	Warehouse 3	3	Customer 3	0	2
W3	Warehouse 3	4	Customer 4	0	1000
W3	Warehouse 3	5	Customer 5	55000	0.5
W3	Warehouse 3	6	Customer 6	0	1.5
W4	Warehouse 4	1	Customer 1	0	1000
W4	Warehouse 4	2	Customer 2	0	1000
W4	Warehouse 4	3	Customer 3	40000	0.2
W4	Warehouse 4	4	Customer 4	0	1.5
W4	Warehouse 4	5	Customer 5	0	0.5
W4	Warehouse 4	6	Customer 6	0	1.5
Total Cost('000)					198.5

6 Conclussion

The data in table 5.1 show that the model has determined that, this distribution pattern costs Rs. 198500 per month. Warehouse capacity is exhausted at warehouse 2 and warehouse 3. Warehouse capacities can be altered within certain limits. For the not fully utilized warehouses 1 and 4 changing capacity with these limits has no effect on the optimal solution. Finally, we conclude that the proposed model is appropriate for the minimizing distribution cost of the study area.

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